Desire-as-Belief Implies Opinionation or Indifference *

Horacio Arló Costa, John Collins, and Isaac Levi

Rationalizations of deliberation often make reference to two kinds of mental state, which we call belief and desire. It is worth asking whether these kinds are necessarily distinct, or whether it might be possible to construe desire as belief of a certain sort — belief, say, about what would be good. An expected value theory formalizes our notions of belief and desire, treating each as a matter of degree. In this context the thesis that desire is belief might amount to the claim that the degree to which an agent desires any proposition A equals the degree to which the agent believes the proposition that A would be good. We shall write this latter proposition 'A°' (pronounced 'A halo'). The Desire-as-Belief Thesis states, then, that to each proposition A there corresponds another proposition A°, where the probability of A° equals the expected value of A.

In 'Desire as Belief,' David Lewis presented an argument against this anti-Humean proposal.[1] Lewis proved that, on pain of triviality, the Desire-as-Belief Thesis cannot be added to the axioms of expected value theory. Our aim in this paper is to present a simpler proof of Lewis's result. The proof we shall give makes clear that the result is a purely synchronic one that depends in no way on the properties of Jeffrey conditionalization or any other revision method.

We shall start by presenting a version of expected value theory like the one developed by Richard Jeffrey.[2] Let X be a Boolean algebra of propositions, with greatest and least elements T and F. An *expected value model* on X is

*This paper appeared in *Analysis*, 55.1, January 1995, pp.2–5.

[1] *Mind* (1988) pp.323–33. John Collins presents an argument against a non-quantitative version of the Thesis in 'Belief, Desire, and Revision,' *Mind* (1988) pp.333–342.

[2] *The Logic of Decision*, 2nd. ed., University of Chicago Press (1983).

a set $\mathcal{M}$ of pairs $\langle P, V \rangle$, where each P is a probability function over X, and each V is an expected value function defined over $X - \{F\}$. Each pair $\langle P, V \rangle$ represents a potential state of belief and desire for the agent. The pairs must satisfy the following four axioms:

Axiom 1: $P(A) \geq 0$.

Axiom 2: $P(T) = 1$.

Axiom 3: If $A\&B = F$, then $P(A \vee B) = P(A) + P(B)$.

Axiom 4: If $P(A\&B) = 0$ and $P(A \vee B) \neq 0$, then

$$V(A \vee B) = \frac{P(A)V(A) + P(B)V(B)}{P(A) + P(B)}$$

We shall be investigating the prospects of adding to this set of axioms the following version of the Desire-as-Belief Thesis:

DAB: For each proposition A, there is a corresponding proposition A° such that for all $\langle P, V \rangle \in \mathcal{M}$, $V(A) = P(A^\circ)$.

Lewis's proof goes roughly like this. Consider an agent with probability and expected value functions $\langle P, V \rangle$ forced to revise by changing the probability of some proposition E to $P(E)+x$ for various values of x. Under Jeffrey conditionalization the posterior probability of the proposition A° is the product of the prior probability of that proposition by some linear function of x, while the posterior expected value of the proposition A is the product of its prior expected value by the quotient of two linear functions of x. It follows that for any given proposition A, $P(A^\circ)$ and $V(A)$ will, in contradiction to the Desire-as-Belief Thesis, be shorn apart by probability kinematics unless either the probability or the expected value of A is unchanged by the revision. But it is absurd to suppose that the probability and expected value of a proposition cannot change simultaneously. Lewis concludes that the Desire-as-Belief Thesis must be rejected.

But there is a more straightforward problem with adding the Thesis to the theory of expected value.

The model $\mathcal{M}$ consists of all the agent's potential belief-desire states. It should include not only the agent's current state $\langle P, V \rangle$, but also any possible state that an agent is rationally permitted to enter in response to changed epistemic circumstances. For example, as a result of learning the truth of some proposition A, the agent's subjective probability and expected value functions might change from P to P_A and from V to V_A. A characteristically Bayesian thought is that this should occasion no significant change in those functions over the subalgebra of propositions that entail A. Of course the probability function must be renormalized when the probability of A is set equal to that of T, if Axiom 2 is to be satisfied. That is to say, $P_A(A\&B) = k.P(A\&B)$ for all B, where $k = 1/P(A)$. P_A is then said to be obtained from P by conditionalization. In the case of the V-function, however, there is no need to make any change at all in assignments to propositions of the form $A\&B$.

These shifts are often held to be rationally obligatory when A is discovered to be true. We make no such claim. We only say that such changes should not be *prohibited* by considerations of rationality. So as to ensure that such shifts are not prohibited, we shall assume that any expected value model also satisfies the following requirement:

Axiom 5: If $\langle P, V \rangle \in \mathcal{M}$ and $P(A) > 0$ then $\exists \langle P_A, V_A \rangle \in \mathcal{M}$ such that for all B

$$\begin{array}{ll} \text{(i)} & P_A(B) = P(B/A), \\ \text{(ii)} & V_A(B) = V(A\&B). \end{array}$$

It must be emphasized that Axiom 5 is not a diachronic principle of change in either probability or expected value judgments. Axiom 5 is purely synchronic.[3] It merely states that if $\langle P, V \rangle$ is a possible belief-desire pair for

[3]Clause (i) is a condition on the agent's confirmational commitments. It is a presupposition of the purely synchronic principle of *confirmational conditionalization* which is to be contrasted with the diachronic principle of *temporal credal conditionalization*. See Isaac Levi, 'On Indeterminate Probabilities,' *The Journal of Philosophy* (1974) pp.391–418 and *The Enterprise of Knowledge*, MIT Press (1980), Section 4.3. In a parallel manner, clause (ii) is a condition on the agent's valuational commitments. Jeffrey discusses it in *The Logic of Decision* at pp.90 and 93. Lewis also imposes this condition on valuational commitment. He refers to clause (ii) in the context of Jeffrey Conditionalization as the *Invariance Assumption*.

the agent at a particular time t, then at that same time t certain other pairs should also count as potential states of belief and desire for the agent.

Our argument depends on two lemmas.

Lemma 1: If $P(A\&B) > 0$ then $V(A\&B) = P(A^\circ/B)$.

Proof: By Axiom 5, $V(A\&B) = V_B(A)$, which, by the Desire-as-Belief Thesis, equals $P_B(A^\circ)$, i.e. $P(A^\circ/B)$.

Lemma 2: If $P(A\&B) > 0$, $P(A\&\neg B) > 0$, and $P(B/A) \neq P(B)$ then $V(A\&B) = V(A\&\neg B)$.

Proof: $P(A^\circ) = P(A^\circ/B)P(B) + P(A^\circ/\neg B)P(\neg B)$ which, by Lemma 1, equals $V(A\&B)P(B) + V(A\&\neg B)P(\neg B)$. Axiom 4 implies that $V(A)$ can be written as $V(A\&B)P(B/A) + V(A\&\neg B)P(\neg B/A)$, and by the Desire-as-Belief Thesis this must also equal $P(A^\circ)$. Hence $V(A\&B)(P(B/A) - P(B))$ equals $V(A\&\neg B)(P(\neg B) - P(\neg B/A))$ and since $P(B/A) - P(B) = P(\neg B) - P(\neg B/A)$, we may divide both sides of the equation by this term if it is non-zero, obtaining $V(A\&B) = V(A\&\neg B)$.

Lemma 2 leads directly to our main result.

The Opinionation or Indifference Theorem: Suppose that the agent's belief-desire state is represented in an expected value model that satisfies Axioms 1–5, and the Desire-as-Belief Thesis. Then, provided that three pairwise inconsistent propositions receive non-zero probability, the agent must view with indifference any proposition whose probability is greater than zero.[4]

Proof: Let the agent's state be given by the pair $\langle P, V \rangle$. If $P(A) = 1$ then $V(A) = V(T)$ is immediate from Axiom 4, so we may assume that $0 < P(A) < 1$, and it suffices to show that $V(A) = V(\neg A)$. It follows from the hypothesis that either A or $\neg A$ may be partitioned into two propositions B, C to which P assigns non-zero probability. Without loss of generality, suppose this is true for $\neg A$. Then $P(A/A \vee B) \neq P(A)$, so by Lemma 2 $V((A \vee B)\&A) = V((A \vee B)\&\neg A))$, i.e. $V(A) = V(B)$. But by a similar

[4]We follow Jeffrey's terminology in saying that the agent is *indifferent to A* iff $V(A) = V(T)$. See *The Logic of Decision* p.82.

argument, $V(A) = V(C)$. Hence $V(\neg A) = V(A)(P(B/\neg A) + P(C/\neg A)) = V(A)$ as required.

The term "opinionated" usually refers to a probability function that assigns probability to a single world only; we are also calling a probability function opinionated if it assigns probability to only two worlds. But even in the slightly looser sense in which we are using the term here, it remains clear that by constraining a theory of expected value to include nothing but opinionated probability functions we trivialize it. We conclude that the Desire-as-Belief Thesis cannot be added to an expected value theory like Jeffrey's.[5]

Department of Philosophy
Columbia University
New York, NY 10027 U.S.A.

[5] We would like to thank Peter Smith for his helpful comments.